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Liquid flow in tubes III

Characteristic data of the transition process

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With 7 figures in the text

ABSTRACT

Qualitative studies on the transition process in tube flow of liquids reported in previous papers of this series have been extended to cover also quantitative relationships of this phenomenon for flow of White Hector bentonite sols of various concentrations.

Experimental evidence is presented which indicates that the transition process depends on some physical—say structural—properties of the liquids which are not accounted for by the coefficient of viscosity and the density. The findings actually demonstrate that the Reynolds law of similarity does not entirely apply to the flows investigated. These findings are supported also by studies of a new transition quantity, *the maximum relative spot velocity* (= maximum relative velocity of the rear of the turbulent slugs as defined previously (Lindgren 1957, pp. 89–98, 147–148)), the determination of which is independent of the viscosity of the liquids.

Introduction

In two papers (Lindgren 1959*a*, *b*), the present author has recently published reports confirming and extending previous findings on the qualitative course of the transition process in tube flow (Lindgren 1954*a*, *b*; 1957). Thus it now may be considered certain that it takes place as the birth and growth of turbulent spots[1] along the tube walls in the same manner as spontaneous transition to turbulence takes place in Blasius flow (flow over flat plates), as observed by other authors (Emmons, 1951, Mitchner, 1954, Schubauer and Klebanoff, 1955). Further it was stressed that the turbulent slugs appearing within the transition region of tube flow should not be compared to the turbulent spots appearing in Blasius flow, the turbulent slugs merely representing such later stages of development of turbulent spots as cannot essentially appear in Blasius flow but only in channel flow (see Lindgren 1959*a*, pp. 112, 115).

The present study is devoted to a quantitative determination of the growth and transport of turbulent slugs within the transition region in relation to previous studies on the same subject.

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[1] Regarding the nomenclature applied, I have to refer to paper II of this series (Lindgren 1959*b* pp. 503–504.

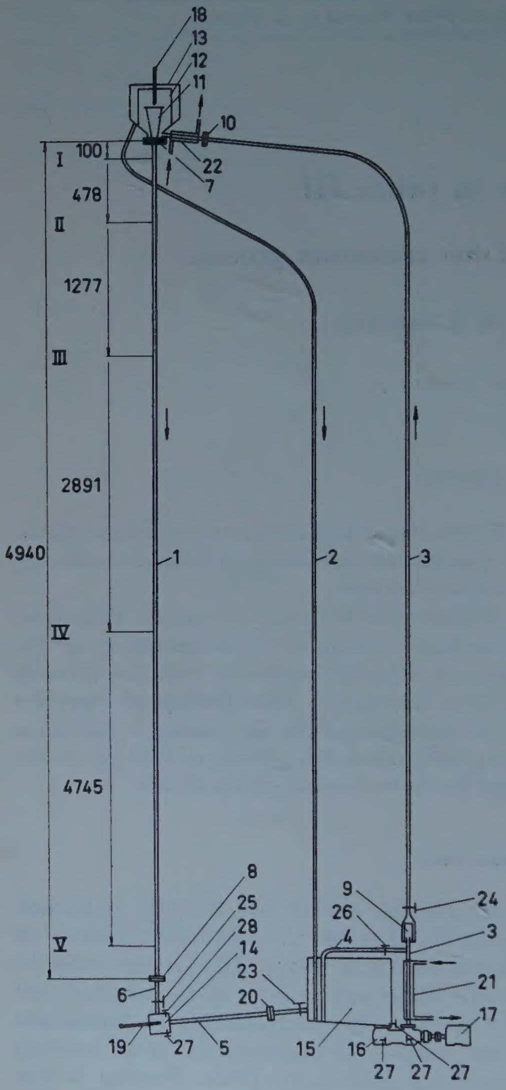

Fig. 1. Arrangement of the experimental flow apparatus

1. Experiment tube
2. Overflow return pipe (PCV plastics)
3. Feeding pipe line (PCV plastics)
4. Short circuit plexiglass pipe
5. 2 measuring pipes
6. Joint hose (PCV plastics)
7. Inlet arrangement from the levelling tank to the experiment tube (see Fig. 2)
8. Coupling flanches
9. Glass wool sieve
10. Glass-fiber-fabric sieve
11. Outlet trumpet from the levelling tank
12. Levelling tank
13. Covering container
14. Calming tank
15. Supply container
16. Circulation pump
17. Electric motor
18, 19. Thermometers
20. 2 housings with measuring orifice plates
21, 22. Heat exchangers
23. 2 regulation cocks
24–26. Stop cocks
27. Tap cocks
28. Bleed cock

As to the previous studies, the stream bi-refringence technique has also been applied in the present investigation. Wite Hector bentonite sols (of distilled water) of various concentrations have been used in the experiments, with special attention to the influence of the bentonite on the transition pattern.

Experimental arrangement

The experimental apparatus used in my previous experiments in this series (Lindgren 1959*a*, *b*), including the same experimental 23.7-mm plexiglass tube, 4.94 m

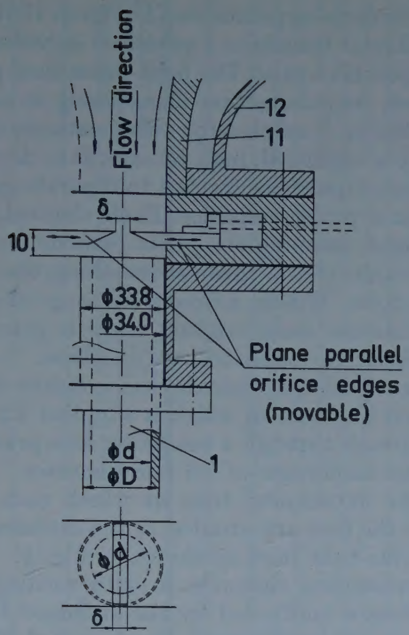

Fig. 2. Arrangement of the experiment tube inlet. 1, experiment tube; 11, outlet trumpet from the levelling tank; 12, levelling tank: d = inner tube diameter = 23,7 mm, D = outer tube diameter = 30 mm. During the present set of experiments the disturbance level of the inlet flow was varied by variation of the width, δ, of the slit between the orifice edges. (Above the coupling flange the outer diameter of the experiment tube is increased to 33.8 mm in order to avoid a "dead" liquid volume.)

length, was used also in the present study and is shown in Fig. 1. Also the inlet arrangement of the experiment tube, by which the disturbance level of the inlet flow is varied, is again shown (Fig. 2) for the convenience of the reader.

The experiments were carried out with White Hector bentonite sols of distilled water of concentrations 0.62, 1.21, 2.40, 4.80 and 9.06‰. The accuracy of the bentonite concentrations determined may be considered better than 1% for the four weakest suspensions and 2% for the 9.06‰ sol. For a description of the process of determination of the bentonite concentrations, reference is made to an earlier paper (Lindgren 1957, pp. 16–17), while brief discussion of the viscosity values applied is given in an Appendix to this paper on p. 112.

Local diameter values of the experiment tube were determined by the process of step-wise filling and emptying of the tube in about 0.3 m lengths. All mean tube diameters thus measured fell within the limits $d = 23.71 \pm 0.06$ mm. The roundness of the tube was checked at four cross-sections at the ends of the tube from which the experiment tube was cut. At these locations the roundness of the tube was kept well within ± 0.05 mm variation in the diameter.

The process of formation, development and transport of turbulent spots and slugs in the flow was simultaneously recorded by means of five optic-electronic devices similar to those used in previous experiments (Lindgren 1957, p. 76) placed along the experiment tube. Each device transmits a polarized light-beam of 0.5 mm diameter across the flow in the experiment tube. The light-beam then passes a stray-light labyrinth via a polaroid sheet, its polarization plane being at an angle of 90° relative to that of the light-beam, before it reaches the photo-cathode of a 9-stage photo-multiplier, which is powered by a stabilized, high-tension, D.C. device giving 1.0 KV of 2‰ stability. Each multiplier output is connected to separate galvanometer channels of an ink-jet oscillograph via a power amplifier. Each channel records the instantaneous intensity of the light reaching the photo-cathode, corresponding possibly to some integrated mean value of the shear stress along the path of the light beam through the flow in the tube. Within a frequency range of up to about 480 fluctuations per second of maximum scale reading there is practically no distortion of the recorded signals, neither in amplitude nor in phase.

The light source of the optic-electronic devices consists of an incandescent lamp with an almost point-sized filament, to which is supplied 1.2 V of 2‰ stability from a D.C. device. The light passes through a condenser lens prior to its passage through the polaroid plate and the diaphragm of 0.5 mm diameter.

The locations along the experiment tube at which each light-beam of the five probing devices traverses the flow are situated at the distances of 0.100, 0.478, 1.277, 2.891 and 4.745 m from the tube inlet as shown in Fig. 1.

Besides the five flow-recording channels, a sixth oscillograph channel has been connected to a time indicator controlled by the standard frequency of the electric mains, printing a mark on the oscillograph paper each 1/50 and 3/10 of a second.

The rate of flow, controlled by the two regulation valves 23 in Fig. 1, is read by either of two strain-gauge pressure transducers, one with a measuring range of $3\text{–}30 \times 10^{-6}$ m^3/s and the other with a range of $20\text{–}200 \times 10^{-6}$ m^3/s. The transducers are supplied by an electric accumulator and are connected to a potentiometer recorder (see Lindgren 1957 pp. 75–76). The rate of flow is indicated by the recorder with an accuracy which has been estimated to be about 2 % over the whole scale. Note that the apparatus is constructed so that calibration of the flow recorder can be performed any time during the run of the experiments without opening the apparatus or changing the flow conditions. Such calibrations have frequently been made during the course of the experiments.

Observations on the appearance and propagation of turbulent slugs in flow of the various bentonite sols

The method of observation applied should allow some quantitative analysis of the eddy structure within turbulent slugs and continuous turbulent flow. For the present, however, I confine myself to some observations with respect to the transition process which do not necessitate a thorough analysis of quantitative relationships between the transmitted signals and the shear structure within the flow at the location of the recording light-beam. It is here sufficient to know that the sensitivity of the recording gauges has been high enough to indicate the passage of large distorted eddies in their final stages of decay in an otherwise laminar flow.

Fig. 3 shows a characteristic record of simultaneously transmitted signals at the

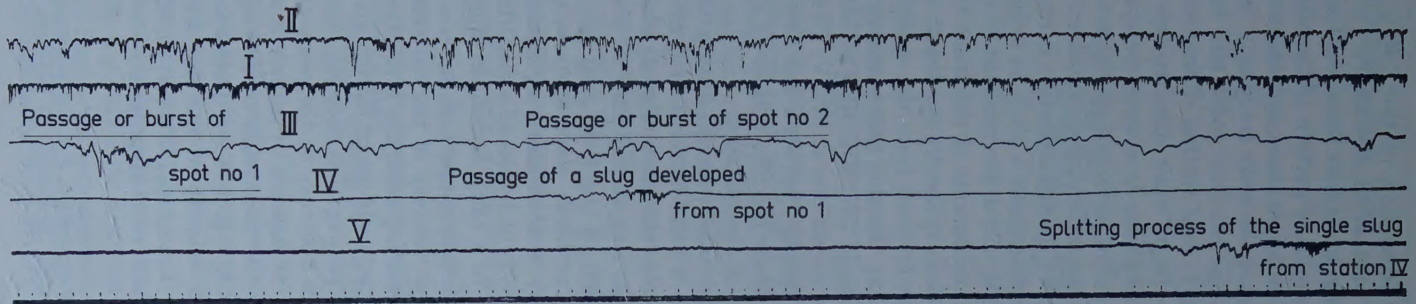

Fig. 3. Oscillogram showing the state of flow of White Hector bentonite sol of 2.4 ‰ concentration at the various observational stations along the 23.71-mm experiment tube shown in Fig. 1.

Trace I (the second trace from the top), representing the flow at Stn I, 4 tube diameters, *d*, from the tube inlet, shows a rather high frequency of the turbulent fluctuations, while already at Stn II, 20 *d* from the tube inlet, trace II shows a noticeable decrease of the frequency pattern indicating that the small entrance eddies dissipate very quickly. At Stn III, 53 *d* from the tube inlet, there remain only some large weak eddies from the entrance turbulence, which always appear to be decaying. However, on two marked locations along trace III we may note the appearance of turbulent spots among the weak inlet eddies. The second spot of the two evidently dissipates before it reaches Stn IV, while the first one has developed into a self-preserving tubulent slug when reaching Stn IV, 122 *d* downstream of the tube inlet, as shown by corresponding trace IV. (Time axis from left to right on the oscillogram.) The slug front appears as some weak deflections of the oscillograph trace from its otherwise straight horizontal course, representing the decaying core eddies brought forward of the self-preserving turbulent slug body or spot, which shows itself by more vigorous amplitude fluctuations. Note also that the slug ceases abruptly at its rear end. The next lowest oscillograph trace, respresenting the flow conditions at Stn V, 200 *d* from the tube inlet, shows that the slug, during its passage from Stn IV to Stn V, is developing into a streak of two units. There appears to be a forward slug body under development within the region of decaying eddies in front of the rear slug body.

The time scale is given from left to right in the lowest oscillograph trace where each division indicates a time interval of 3/10 of a second.

five observational locations along the tube from flow of the 2.40‰ bentonite sol within the transition region, at a Reynolds number of $R \approx 2800$[1].

The second trace from the top in the oscillogram is that from Stn I at 4.2 tube diameters, d, from the tube inlet and shows a frequency pattern of the eddies appearing within the disturbed (turbulent) inlet flow. The upper trace, representing Stn II, 20 d from the tube inlet, shows a distinct decrease of the eddy frequency, indicating that the smallest eddies very quickly dissipate, while the flow could still be looked upon as being fully turbulent. However, at Stn III, 34 d further downstream, the third oscillograph trace indicates that only some large weak disturbances remain from the intensive turbulent inlet flow, which turbulence is always decaying. However, here and there the trace indicates the appearance of turbulent spots among the damped inlet eddies. Two such occurrences of spots are indicated on horizontal lines above this trace. The second of the two spots evidently dissipates before it reaches the next observational station, while the first one develops into a slug, as indicated in trace IV, representing the flow conditions at Stn IV, 122 d downstream of the tube inlet. We note that the slug first appears as weak deflections of the trace from its otherwise straight horizontal course. These deflections evidently represent the passage of the dissipating core eddies which are brought forward from the more vigorous self-preserving body of the slug. We also note that the slug ceases abruptly at its rear end, as previously pointed out (Lindgren 1954*a* p. 229, 301; 1957 p. 35; 1959 p. 118). The next lowest trace, representing the flow conditions at Stn V, 78 d downstream of Stn IV, shows that the slug, during its passage between the two stations, has developed a tendency to split into two units. There is a new slug body under development within the disturbed flow region ahead of the old spot.

The lowest oscillograph trace indicates the time passage during the run of the oscillograms. Each scale division represents a time-interval of $\frac{3}{10}$ of a second synchronized with the standard frequency of the electric mains. Noting the time interval between the passage of the front and rear of the slug at, say Stns IV and V, it is thus possible to determine the speed of propagation of the front and rear of the slugs. The determination of the rear velocity is no problem, since the rear of the slugs is always pretty well defined. The front velocity is determined by noting the passage of the forward part of the vigorously eddying body or bodies of a slug. (It is, of course, of no importance on which part of the front of a fully developed slug the speed of propagation is determined, if only it is a well-defined part of it.)

Although determinations of the propagation velocity of the turbulent slugs may not be as accurate as that of the rear, the measuring accuracy is of higher order than the variations of the front velocity that may actually occur during an experimental run with fixed settings of the apparatus. The propagation velocity of the rear of the slugs is a more consistent quantity which does not vary to the same extent.

Remembering previous statements that the elongation of turbulent streaks during their downstream transport actually implies the formation of new spots ahead of already existing ones (Lindgren 1959*a* p. 113; *b* p. 518), it is not surprising that there are wide variations in the propagation velocity of the slug fronts, since we know from previous and present experiments (Lindgren 1957, pp. 49, 52) that release and development of turbulent spots takes place very irregularly.

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[1] The Reynolds number is here derived from the expression $R = Ud/\nu$, where U = mean flow velocity, d = tube diameter and ν = kinematic viscosity according to previous determinations (Lindgren 1957, pp 9–29). See Appendix.

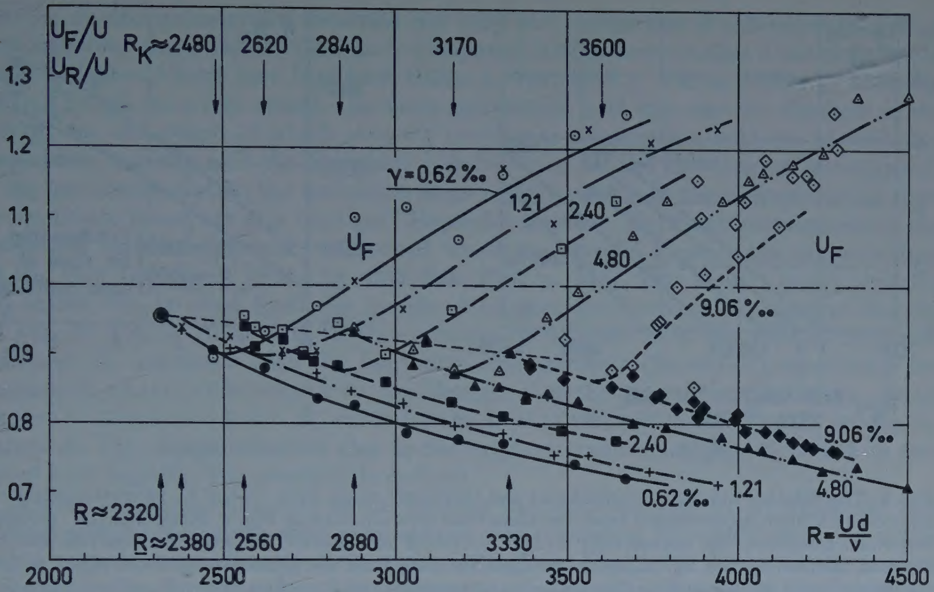

Fig. 4. The results of the experimental investigation on flow of White Hector bentonite sols of various concentration, γ, are collected in this diagram. U_F = propagation velocity of the front of the turbulent slugs, U_R = propagation velocity of rear of the slugs, while U = mean flow velocity, R = Reynolds number, d = tube diameter and ν = kinematic viscosity of the flowing liquid. Note that the rear velocity, U_R, of the slugs is identical with the propagation velocity of the turbulent spots that at the moment constitute the slug.

○ = experimental determination of the front veolocity of turbulent slugs in flow of bentonite sol of concentration, $\gamma = 0.62$ ‰
● = same rear velocity
× = front velocity of same, 1.21 ‰ bentonite sol
+ = rear velocity
□ = front velocity of same, 2.40 ‰ bentonite sol
■ = rear velocity
△ = front velocity of same, 4.80 ‰ bentonite sol
▲ = rear velocity
◇ = front velocity of same, 9.06 ‰ bentonite sol
◆ = rear velocity
$\underline{R}$ = transition Reynolds number. i.e. the least Reynolds number at which self-preserving turbulent slugs ever appear
R_k = "critical" Reynolds number i.e. the Reynolds number above which the slugs begin to split and to extend their lengths.

Fig. 4 contains the collected results of the present experimental findings and shows the average propagation velocity of the front, U_F, and rear, U_R, of the turbulent slugs over the mean flow velocity, U, in relation to the Reynolds number, R, for flows of the various bentonite sols.

Here we should observe that in the light of previous and present studies (Lindgren 1959*a*, p. 113; *b* p. 517–518) the rear velocity of the slugs, U_R, should actually be interpreted to imply the propagation velocity of the self-preserving turbulent spots which in fact constitute the turbulent streaks. As a consequence of this interpretation the ratio of the difference between the front and rear velocities, $U_F - U_R$, to the mean

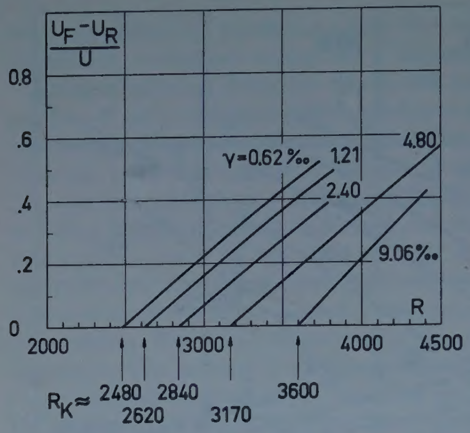

Fig. 5.

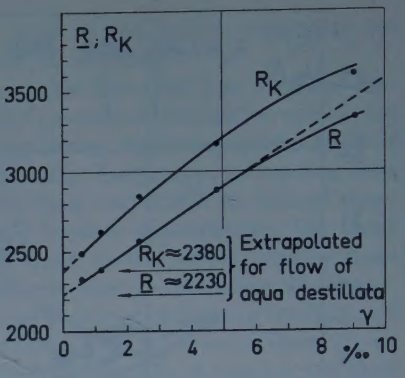

Fig. 6.

Fig. 5. The relative expansion velocity of the turbulent slugs, $(U_F - U_R)/U$, plotted against the Reynolds number, as extracted from the smoothed relationships of Fig. 4. Note that the "critical" Reynolds numbers, R_k, correspond to the points of intersection between the curves and the abscissa. Evidently the R_k values determined increase with the bentonite concentration, γ. (For the definition of R_k, see text to Fig. 4 which also contains the R_k values determined from the above diagrams.)

We remember that the quantity $(U_F - U_R)/U$ may be regarded as representing a relative measure of the formation of new turbulent spots within the instantaneous slug front.

Fig. 6. The transition Reynolds number, $\underline{R}$, which is the smallest Reynolds number at which any self-preserving turbulent slugs appear, plotted against the concentration, γ, of flows of the bentonite sols. $\underline{R}$ appears to increase with γ and extrapolation towards zero concentration gives a $\underline{R}$ value of about 2230 for flow of distilled water under the same flow conditions.

The "critical" Reynolds number, R_k, as read from Fig. 5, is also plotted against γ in this diagram. Extrapolation gives $R_k \approx 2380$ for flow of distilled water which could be compared to the "classical" value $R_k = 2320$.

flow velocity U, *the relative expansion velocity*, $(U_F - U_R)/U$, could be taken as a relative measure of the formation of new spots. Thus the graphs for U_F in Fig. 4 have been smoothed somewhere along the lower limits of the rather large scatter of the experimentally determined values of this quantity. These curves possibly may represent an approximate lower limit of equilibrium for the formation of new turbulent spots. On this basis graphs of the relative expansion velocities, $(U_F - U_R)/U$, as read from the curves of Fig. 4, are shown in Fig. 5 plotted against the Reynolds number, R, of the flow of the various bentonite sols.

We note in Fig. 5 that the quantity $(U_F - U_R)/U$ seems to increase rectilinearily with R in the first part of the flow region where the slugs begin to elongate themselves during their downstream transport. With increasing R, however, the rate of increase of $(U_F - U_R)/U$ decreases for the three sols of lower concentration. We also note that the curves representing the flows of the various sols are shifted toward higher R-values for incresing concentration, γ, and that they appear to be approximately parallel to each other except for the graph for the sol of highest concentration, 9.06 ‰. However, the latter deviation is readily explained by the high velocities necessary to cause transition in this flow. The relative tube length is actually too short, covering not more than the entrance length for establishing the developed laminar velocity profile. In this manner are recorded not only the occurrence of self-preserving slugs but also some slugs which would have dissipated farther downstream if the experiment tube had been longer (see Lindgren 1954*a* p. 304; 1957 p. 146 or 1959*a* p. 113; *b* p. 517). Though, to a less extent, the same statements hold true also for the flow of the 4.80 ‰ sol, the graph of which shows a rectilinear relationship between the relative expansion velocity and the Reynolds number over all the flow region investigated.

As previously stated, the turbulent slugs begin to elongate themselves during their downstream transport at a "critical" Reynolds number, R_k, which corresponds to the "classical" critical Reynolds number at which is observed a rise in the pressure drop of the flow (Lindgren 1954*a* pp. 303–304; 1957 p. 148). This should imply that the R_k-values can be read from the points of intersection between the graphs and the R axis in Fig. 5. According to this notation it appears that the critical Reynolds number, R_k, increases with the bentonite concentration. However, considering the discussion above with respect to the shortness of the experiment tube, we should expect to obtain too low a R_k value for the flow of the 9.06 ‰ sol. The R_k values obtained in Fig. 5 are indicated also in the original diagrams of Fig. 4, which in fact show how vaguely this quantity is defined.

A more consistent quantity is the smallest Reynolds number, $\underline{R}$, at which the first self-preserving turbulent slugs ever appear (the transition Reynolds number), which value may be read directly from the plottings in Fig. 4 for flows of the various bentonite sols. The $\underline{R}$ values also evidently appear to increase with the bentonite concentration, γ, as shown in the diagram of Fig. 6. However, as with the R_k value, we may expect too low a $\underline{R}$ value for the flow of the 9.06 ‰ sol. This is an important point, since, apart from the $\underline{R}$ value for the flow of this sol, there appears to be a rectilinear increase of $\underline{R}$ with γ.

By extrapolation of the graph of the $\underline{R}$ values to zero concentration, we receive an extrapolated transition Reynolds number of $\underline{R} \approx 2230$ for flow of aqua distillata under the same experimental conditions. The same procedure with regard to the critical Reynolds number also plotted in the diagram of Fig. 6 gives a R_k value of about 2380 which should consequently be compared to the "classical" valued $R_k = 2320$.

Thus the present experiments indicate that both the $\underline{R}$ and R_k values increase with increasing γ even for rather low amounts of bentonite. This observation differs from previously reported observations on the transition process in flow through tubes of more varying diameter (Lindgren 1957, p. 84), which observations did not show such an influence on the transition Reynolds number, provided the bentonite concentration was below about 1.2 ‰.

The reliability of the observation that $\underline{R}$ and R_k vary with γ is supported by the results of the slug velocity measurements plotted in Fig. 4. Noting that the relative spot velocity, U_R / U, has a maximum value, $(U_R / U)_{\max}$, at the smallest Reynolds number, $\underline{R}$, at which the self-preserving slugs ever appear, we observe that the $(U_R / U)_{\max}$ value for flow of the various bentonite sols decreases with the concentration, γ, along the thin dotted line which smoothly joins the values measured. This observation is noteworthy, because the quantity U_R / U—in contrast to the Reynolds number—does not formally involve the viscosity of the liquids. The variation of $(U_R / U)_{\max}$ with γ is shown in Fig. 7.

In order to check whether the development of the velocity profile could possibly have influenced the $(U_R / U)_{\max}$ relationships obtained, a numerical check was made by using the qunatity $(U_R / \overline{U})_{\max}$ instead (here $\overline{U}$ = the flow velocity at the tube centre). No noticeable change of the course of the relationships obtained was noted,

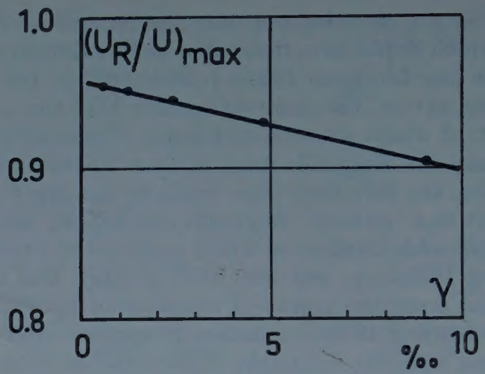

Fig. 7. The maximum relative turbulent spot velocity $(U_R/U)_{max}$ (=the maximum relative propagation velocity of the rear of the turbulent slugs) plotted against the bentonite concentration, γ, of flows of the various sols.

$(U_R/U)_{max}$, which is a quantity that corresponds to the transition Reynolds number $\underline{R}$, evidently decreases with increasing γ, i.e. increasing $\underline{R}$ values. This is a rather remarkable relationship since it means that a pertinent transition quantity, which is formally independent of the viscosity of the flowing liquids and which measurement is a purely mechanical process, varies with some at present unknown physical properties of the flowing liquids.

except, of course, for a parallel shift of the curves. (In fact, the maximum deviation possible from the parallel shift was calculated to be less than 3 % for flow of the 9.06 ‰ bentonite sol around the transition Reynolds number, which should be the critical case.)

Of course, we cannot a priori exclude a possibility that the inlet disturbances may depend on the bentonite concentration of the flowing sols and thus influence the conditions for flow breakdown and bursts of turbulent spots. This state of affairs, however, does not concern the experimental findings reported here, since under the circumstances turbulent spots spring into existence for lower Reynolds numbers than the $\underline{R}$ values reported. These spots or slugs, however, are not self-preserving, but fade away after travelling some distance, while the transition quantities $\underline{R}$ and $(U_R/U)_{max}$ determine quantitative relationships of limits for self-preservation of turbulent slugs once they are formed. That this is so comes from the fact that both $\underline{R}$, R_k and $(U_R/U)_{max}$ for flows of the various sols were received for an opening of the disturbance slit at the tube inlet of about $\delta = 6$ mm (see Fig 2) and did not change for a further decrease of its width (increase of the disturbance level) until $\delta = 2 - 1$ mm whereafter cavitation appeared in some of the flows. This is in agreement with already since long wellknown conditions that the numerical values of the transition quantities $\underline{R}$ and R_k are obtained for some least disturbance level but are not influenced by a further increase of it. (See Lindgren 1954 *a*, p. 295; 1957 pp. 6, 33–36, 99–101, 146.)

Conclusions

From the experimental findings reported we may conclude that at least for flow of betonite sols there must be some other physical characteristic of the flowing liquid which, besides viscosity, density and mean flow velocity, influences the transition process. This means that the Reynolds number alone is not sufficient to describe the conditions determining the onset turbulence in tube flow.

In this connection I want to draw attention to the fact that the systematic variation of the maximum relative spot velocity, $(U_R/U)_{\max}$ with the bentonite concentration, γ, implies, that a pertinent transition quantity the determination of which does not involve the viscosity, has been found to vary with some, at present, unknown physical properties of the flowing liquids.

It is important to note that present and previous experiments indicate that the rear propagation velocity of the slugs, U_R, actually means the propagation velocity of the turbulent spots that constitute the slug, while the ratio of difference between the front velocity, U_F, and rear velocity, U_R, of the slugs to the mean flow velocity, U, may be taken as a relative measure of the formation of new spots at the instantaneous slug front.

ACKNOWLEDGEMENTS

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The experiments have been performed within the administration of the Division of Mechanics I (Head: Professor O. H. Faxén) and on laboratory premises belonging to the Division of Hydraulics Engineering (Head: Professor Magnus Oledal).

As usual I have had the advantage of having the manuscript revised by Professor Hilding Faxén, whose guidance I have enjoyed during so many years of research. Professor Sune Berndt has also treated me roughly, making some pertinent remarks and it has been of special value to me that this time also my friend, Professor Mahinder Uberoi of Michigan University, Ann Arbor, has scrutinized the manuscript prior to its printing.

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APPENDIX

Preparation of the bentontite suspensions, determination of their viscosity and discussion of the evaluation of the experiments

Considering that complications may arise when determining the viscosity of sols, much time and work was previously spent (Lindgren 1957, pp. 9–29) in order to determine applicable relationships between "liquid viscosity", $\mu_\infty(\gamma,t)$, and temperature, t, for White Hector bentonite sols of various concentration, γ. The "liquid viscosity", μ_∞, is here defined as the smallest viscosity value obtained when applying rates of deformation large enough to break down the gel-structure of the sol at rest and no further decrease in μ occurs for still-increasing rates of shear.

The measurements revealed exponential relationships between μ_∞ and γ according to the formula

$$\mu_\infty(\gamma,t) = \mu_w(t) \cdot 10^{0{,}0314 \cdot \gamma}$$

where $\mu_w(t)$ = standard viscosity of distilled water at temperature, t, according to Bingham & Jackson (1918)
and γ = bentonite concentration in ‰.

Knowing the density of water and bentonite suspensions, it is then easy to obtain any kinematic viscosity values desired.

The viscosity values of water due to Bingham & Jackson (1918) were used both as calibration and reference standards in the process of determination of the viscosity of the bentonite sols used.

Of course, the formula arrived at can be valid only for a certain type of bentonite sols prepared in a certain manner. The method used for preparation of the bentonite sols has been described previously (Lindgren 1957, pp. 11–12).

In order to apply the liquid viscosity values determined for evaluation of flow experiments, it is, of course, necessary to provide for a complete breakdown of the gel-structure of the sols also in the flow experiments, otherwise it is of no use however carefully the viscosity determinations are made.

Fortunately the present experiments automatically seem to fulfill the above conditions, since the investigation of the transition process necessarily implies high disturbance levels at the tube inlet for low rates of flow, and high rates of flow in case of low disturbance levels at the tube inlet. In both cases this means high rates of shear, probably much higher than necessary for breaking down the gel-structure of the sols, with no possibility of re-establishment of the structure prior to the occurrence of flow break-down initiating the bursts of turbulent spots and their subsequent development into turbulent slugs.

Tryckt den 8 juli 1959

Uppsala 1959. Almqvist & Wiksells Boktryckeri AB